

THE EFFECT OF STRATIFICATION ON THE ROUGHNESS LENGTH AND DISPLACEMENT HEIGHT

S. S. Zilitinkevich^{1,2,3}, I. Mammarella^{1,2},
A. Baklanov⁴, and S. M. Joffre²

1. Atmospheric Sciences, University of Helsinki, Finland
2. Finnish Meteorological Institute, Helsinki, Finland
3. Nansen Environmental and Remote Sensing Centre /
Bjerknes Centre for Climate Research, Bergen, Norway
4. Danish Meteorological Institute, Copenhagen, Denmark



Reference

S. S. Zilitinkevich, I. Mammarella, A. A. Baklanov, and S. M. Joffre, 2008: The effect of stratification on the roughness length and displacement height. *Boundary-Layer Meteorology*. In press.



Content

- Roughness length and displacement height:

$$u(z) = \frac{u_*}{k} \left[\ln \frac{z - d_{0u}}{z_{0u}} + \Psi_u \left(\frac{z}{L} \right) \right]$$

- No stability dependence of z_{0u} (and d_{0u}) in engineering fluid mechanics: neutral-stability $z_0 = \text{level}$, at which $u(z)$ plotted vs. $\ln z$ approaches zero; $z_0 \sim 1/25$ of typical height of roughness elements, h_0

- Meteorology / oceanography: h_0 comparable with MO length $L = \frac{u_*^3}{-\beta F_{\theta s}}$

- Stability dependence of the actual roughness length, z_{0u} :

$z_{0u} < z_0$ in stable stratification; $z_{0u} > z_0$ in unstable stratification



Surface layer and roughness length

Self similarity in the surface layer (SL)	$5h_0 < z < 10^{-1} h$
Height-constant fluxes:	$\tau \approx \tau _{z=5h_0} \equiv u_*^2$
u_* and z serve as turbulent scales:	$u_T \sim u_*, l_T \sim z$
Eddy viscosity ($k \approx 0.4$)	$K_M (\sim u_T l_T) = k u_* z$
Velocity gradient	$\partial U / \partial z = \tau / K_M = u_* / kz$
Integration constant:	$U = k^{-1} u_* \ln z + \text{constant} = k^{-1} u_* \ln(z / z_{0u})$
z_{0u} (redefined constant of integration) is "roughness length"	
"Displacement height" d_{0u}	$U = k^{-1} u_* \ln[(z - d_{u0}) / z_{u0}]$
Not applied to the roughness layer (RL) $0 < z < 5h_0$	



Parameters controlling z_{0u}

Smooth surfaces: viscous layer $\rightarrow z_{0u} \sim \nu / u_*$

Very rough surfaces: pressure forces depend on:
obstacle height h_0
velocity in the roughness layer $U_R \sim u_*$

$z_{0u} = z_{0u}(h_0, u_*) \sim h_0$ (in sand roughness experiments $z_{0u} \approx \frac{1}{30} h_0$)

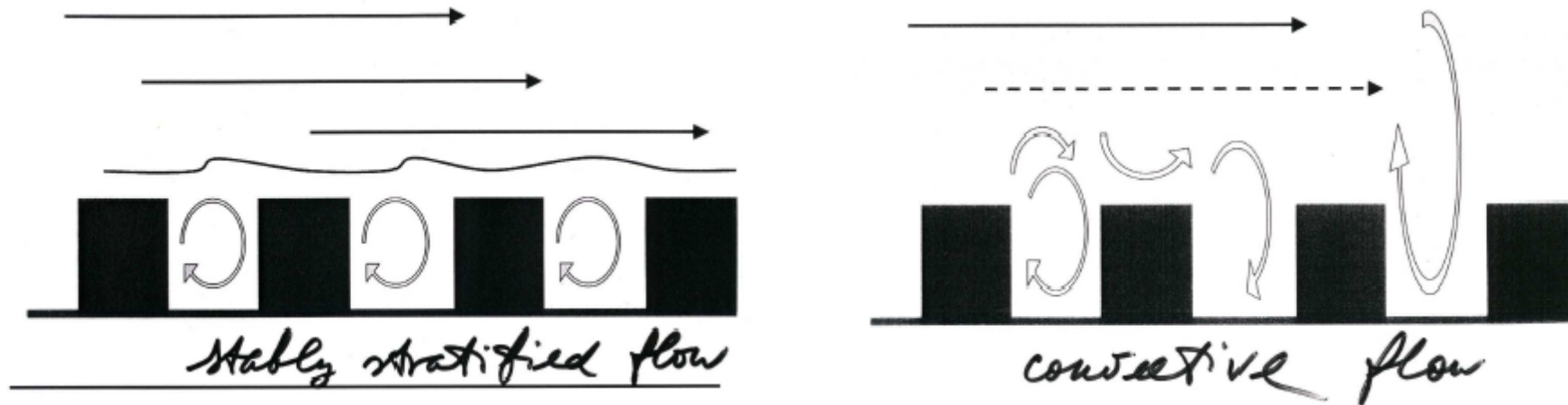
No dependence on u_* ; surfaces characterised by $z_{0u} = \text{constant}$

Generally $z_{0u} = h_0 f_0(\text{Re}_0)$ where $\text{Re}_0 = u_* h_0 / \nu$

Stratification at M-O length $L = -u_*^3 F_b^{-1}$ comparable with h_0



Stability Dependence of Roughness Length



For urban and vegetation canopies with roughness-element heights (20-50 m) comparable with the Monin-Obukhov turbulent length scale, L , the surface resistance and roughness length depend on stratification

Background physics and effect of stratification

Physically z_{0u} = depth of a sub-layer within RL ($0 < z < 5h_0$)
with 90% of the velocity drop from $U_R \sim u_*$ (approached at $z \sim h_0$)

From $\tau = K_{M(RL)} \partial U / \partial z$, $\tau \sim u_*^2$ and $\partial U / \partial z \sim U_R / z_{0u} \sim u_* / z_{0u}$

$$z_{0u} \sim K_{M(RL)} / u_*$$

$K_M(RL) = K_M(h_0 + 0)$ from matching the RL and the surface-layer

Neutral: $K_M \sim u_* h_0 \Rightarrow$ classical formula $z_{0u} \sim h_0$

Stable: $K_M = k u_* z (1 + C_u z / L)^{-1} \sim u_* L \Rightarrow z_{0u} \sim L$

Unstable: $K_M = k u_* z + C_U^{-1} F_b^{1/3} z^{4/3} \sim F_b^{1/3} z^{4/3} \Rightarrow z_{0u} \sim h_0 (-h_0 / L)^{1/3}$



Recommended formulation

Neutral \Leftrightarrow stable $\frac{z_{0u}}{z_0} = \frac{1}{1 + C_{SS} h_0 / L}$

Neutral \Leftrightarrow unstable $\frac{z_{0u}}{z_0} = 1 + C_{US} \left(\frac{h_0}{-L} \right)^{1/3}$

Constants: $C_{SS} = 8.13 \pm 0.21$, $C_{US} = 1.24 \pm 0.05$



Experimental datasets



Sodankyla Meteorological Observatory, Boreal forest (FMI)

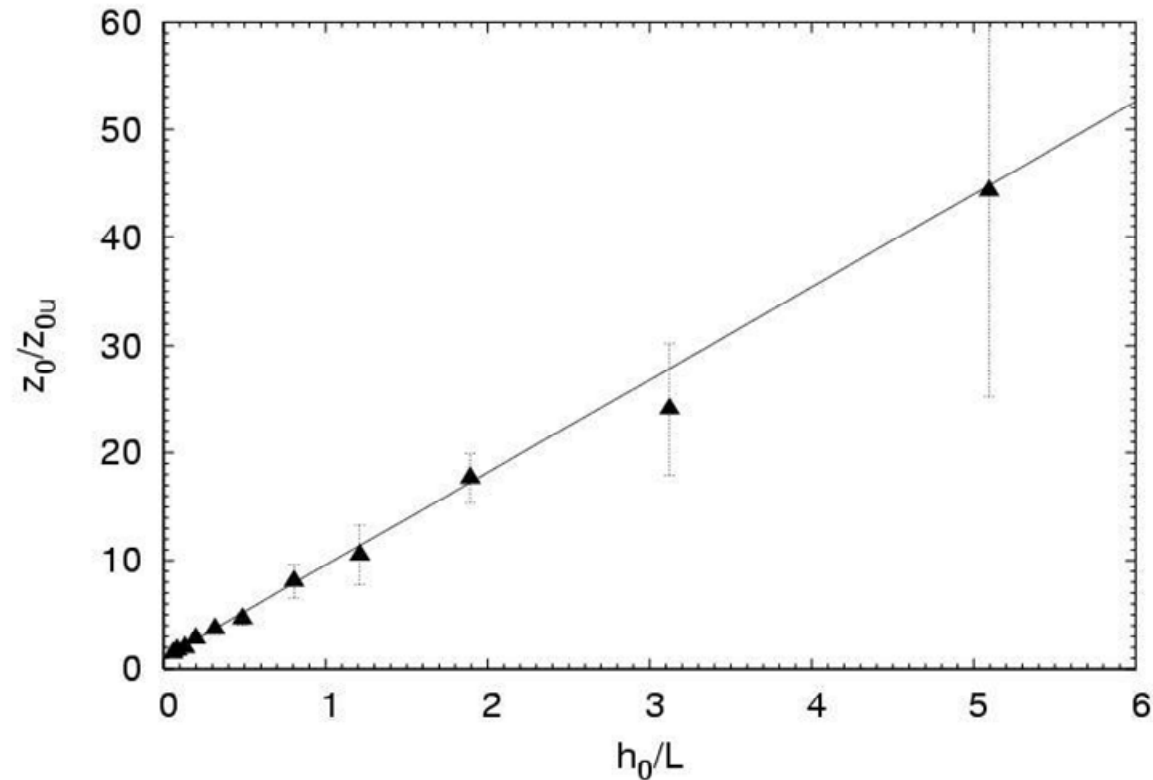
$h \approx 13$ m, measurement levels 23, 25, 47 m



BUBBLE urban BL experiment, Basel, Sperrstrasse (Rotach et al., 2004)

$h \approx 14.6$ m, measurement levels 3.6, 11.3, 14.7, 17.9, 22.4, 31.7 m

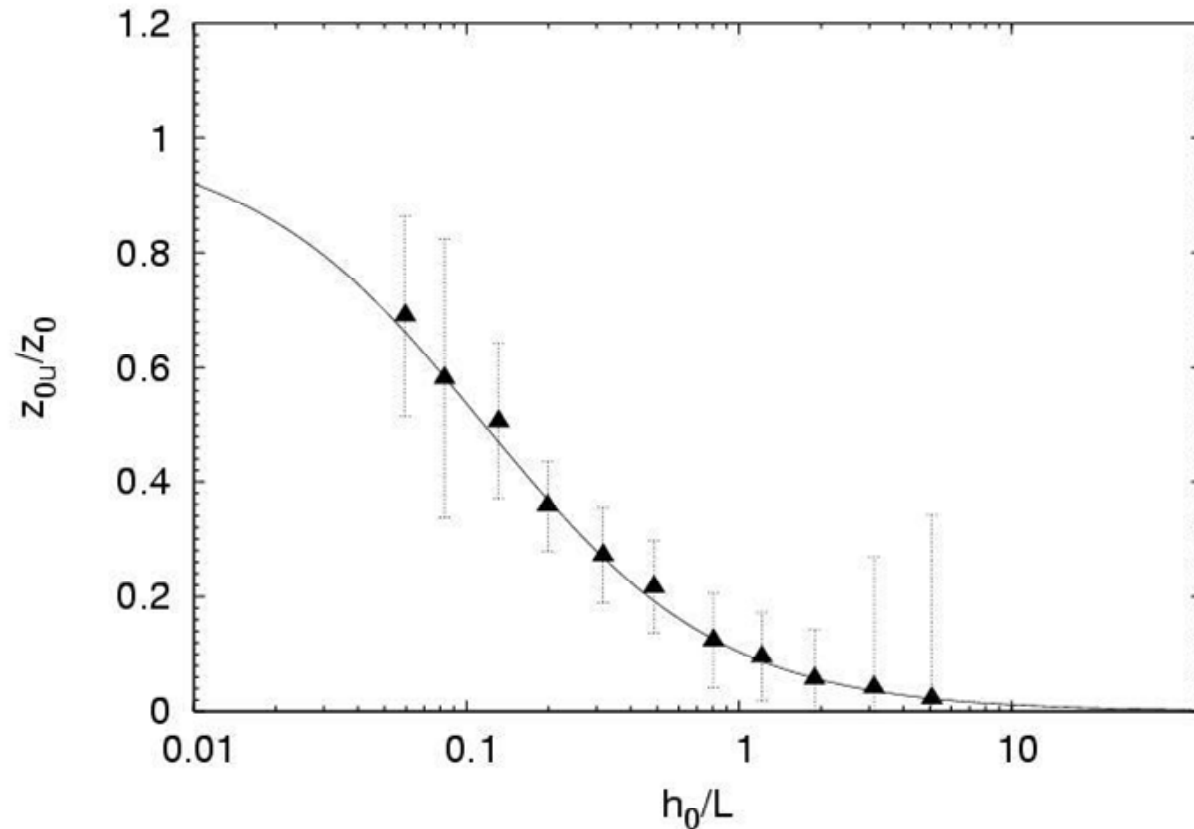
Stable stratification



Bin-average values of z_0 / z_{0u} (neutral- over actual-roughness lengths) versus h_0/L in stable stratification for Boreal forest ($h_0=13.5$ m; $z_0=1.1\pm 0.3$ m). Bars are standard errors; the curve is $z_0 / z_{0u} = 1 + 8.13h_0 / L$.



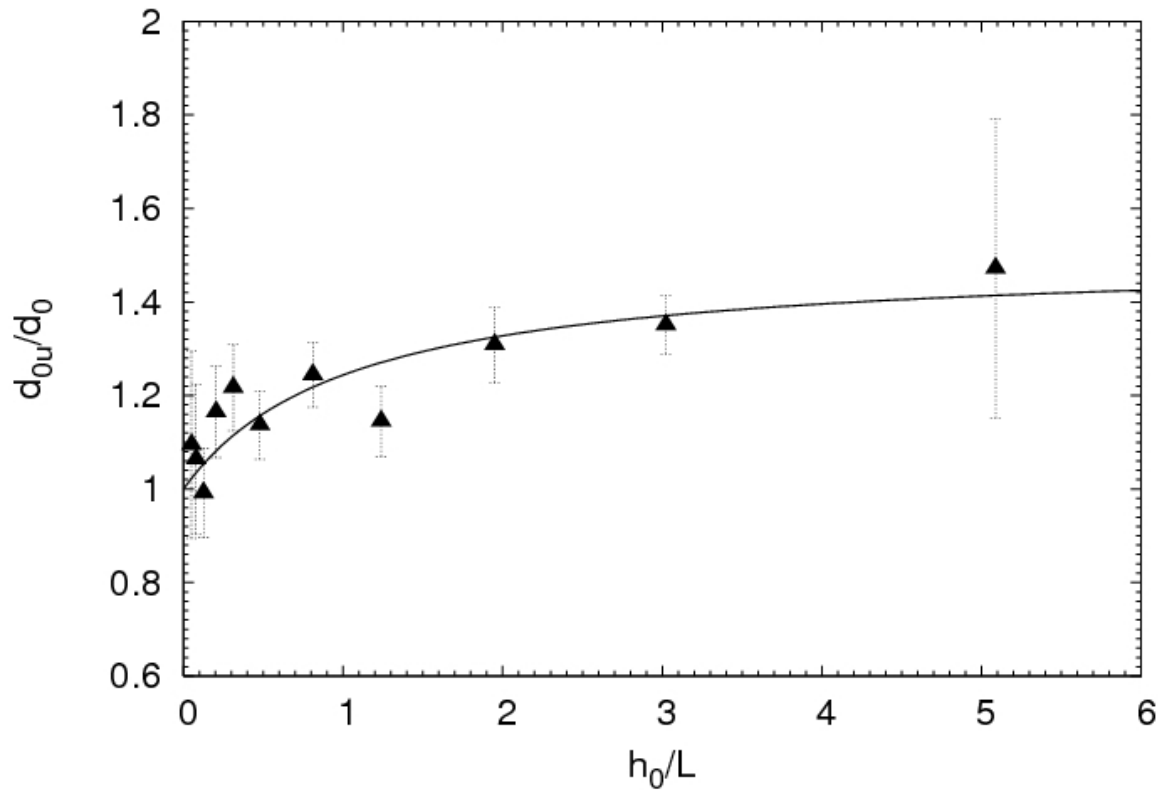
Stable stratification



Bin-average values of z_{0u}/z_0 (actual- over neutral-roughness lengths) versus h_0/L in stable stratification for boreal forest ($h_0=13.5$ m; $z_0=1.1\pm 0.3$ m). Bars are standard errors; the curve is $z_{0u}/z_0 = (1 + 8.13h_0/L)^{-1}$.



Stable stratification



Displacement height over its neutral-stability value in stable stratification.
Boreal forest ($h_0 = 15$ m, $d_0 = 9.8$ m).

The curve is $d_{0u} / d_0 = 1 + 0.5(h_0 / L)(1.05 + h_0 / L)^{-1}$

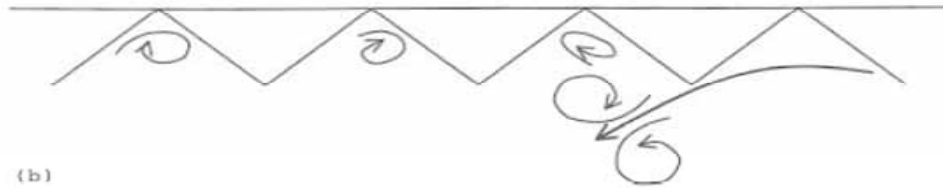
Unstable stratification

Convective eddies extend in the vertical causing $z_0 > z_{0u}$

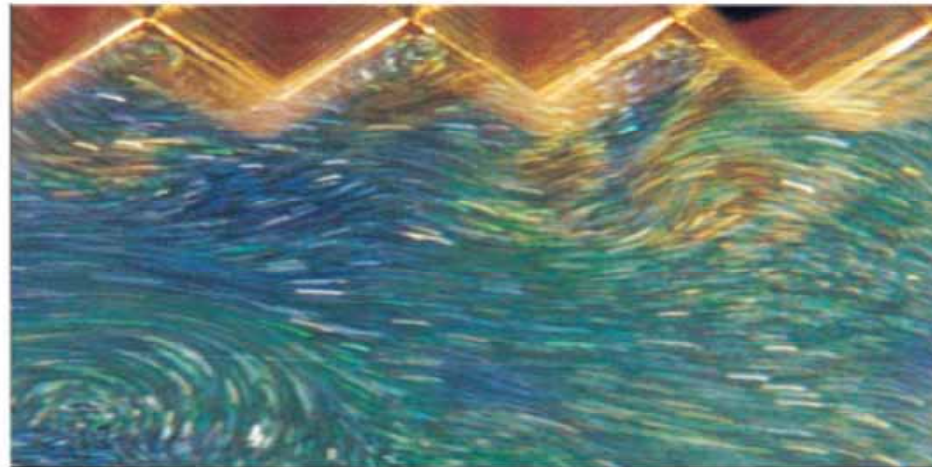
VOLUME 81, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1998

Y.-B. Du and P. Tong, Enhanced Heat Transport in Turbulent Convection over a Rough Surface

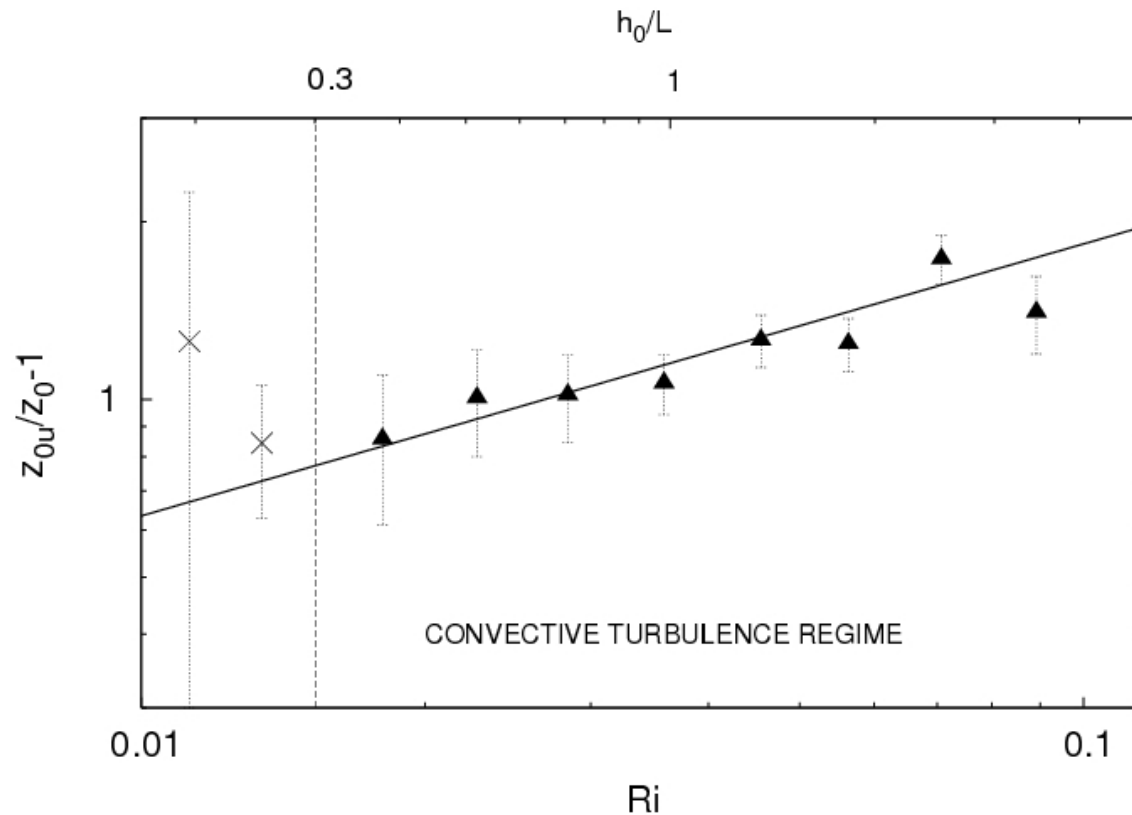
(a)



(b)



Unstable stratification

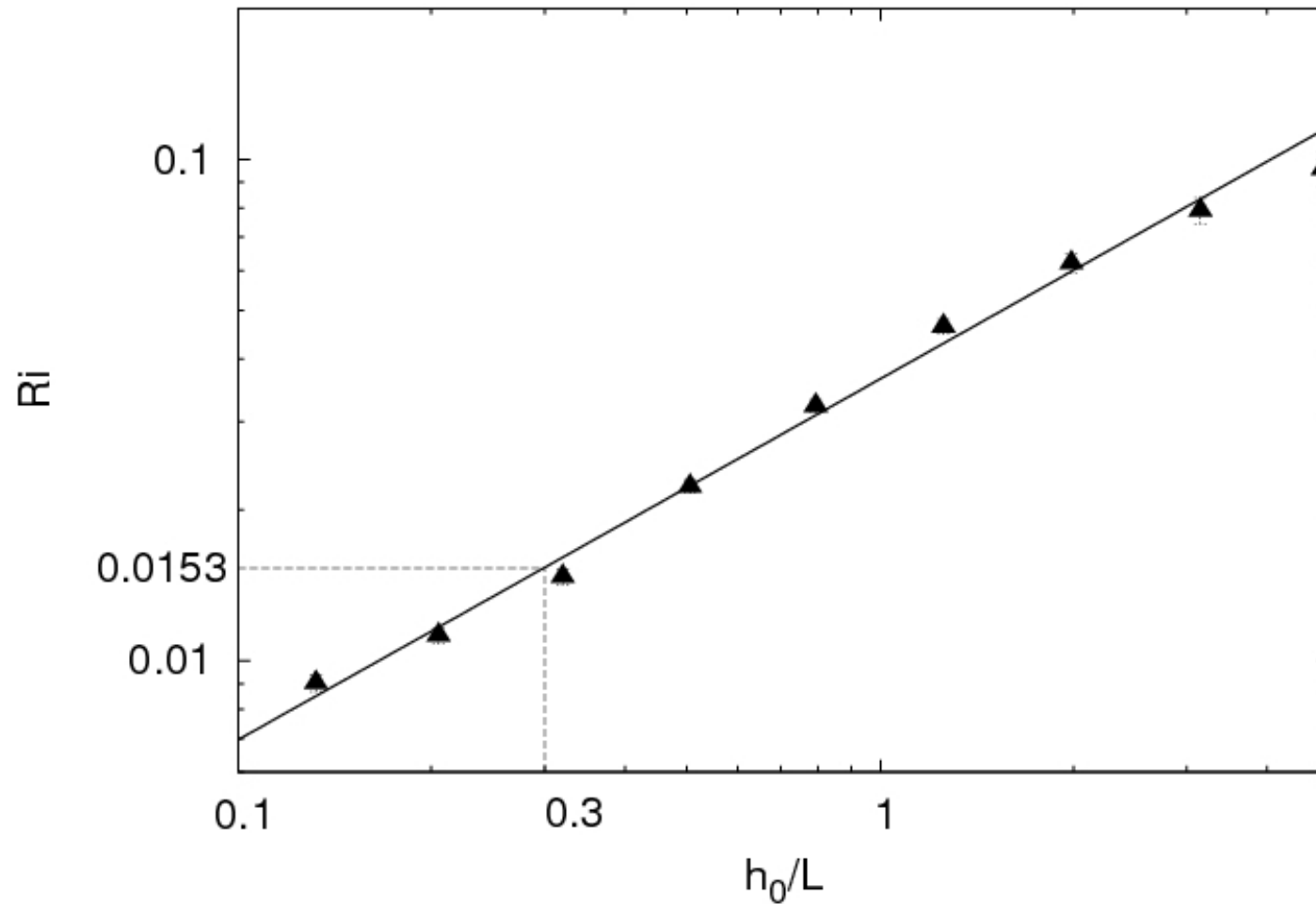


Unstable stratification, Basel, z_0'/z_{0u} vs. $Ri = (gh_0/\Theta_{32})(\Theta_{18}-\Theta_{32})/(U_{32})^2$
 Building height = 14.6 m, neutral roughness $z_0 = 1.2$ m; BUBBLE, Rotach et al., 2005).

h_0/L through empirical dependence on Ri on (next figure)

The curve ($z_0'/z_{0u} = 1 + 5.31Ri^{6/13}$) confirms theoretical $z_{0u}/z_0 = 1 + 1.15(h_0/L)^{1/3}$

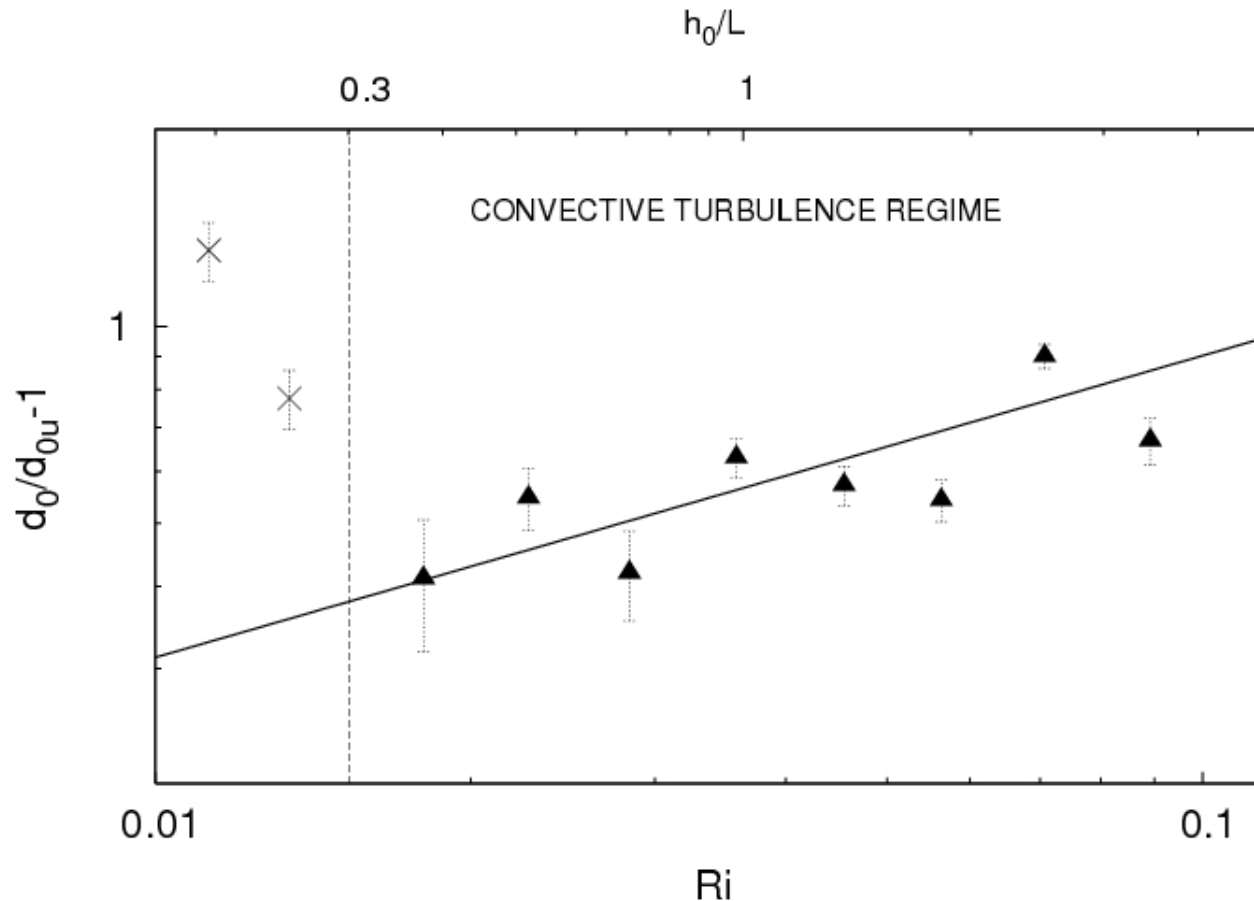
Unstable stratification



$$\text{Empirical Ri} = 0.0365 (h_0/-L)^{13/18}$$



Unstable stratification



Displacement height in unstable stratification (Basel): $d_0 / d_{ou} - 1$ versus Ri

The line confirms theoretical dependence:
$$d_{ou} = \frac{d_0}{1 + C_{DC} (h_0 / -L)^{1/3}}$$

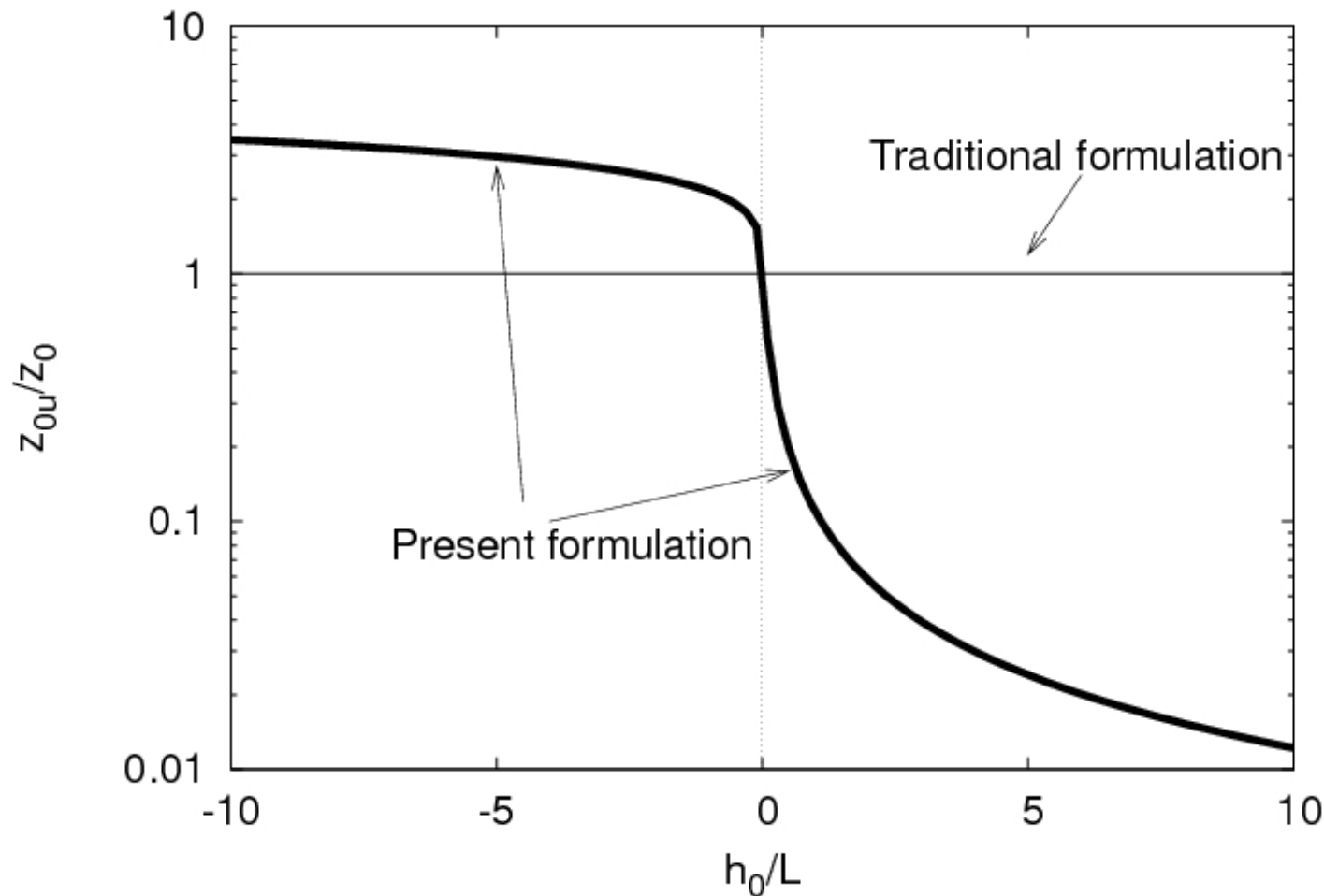


STABILITY DEPENDENCE OF THE ROUGHNESS LENGTH

in the "meteorological interval" $-10 < h_0/L < 10$ after new theory and experimental data

Solid line: z_{0u}/z_0 versus h_0/L

Thin line: traditional formulation $z_{0u} = z_0$

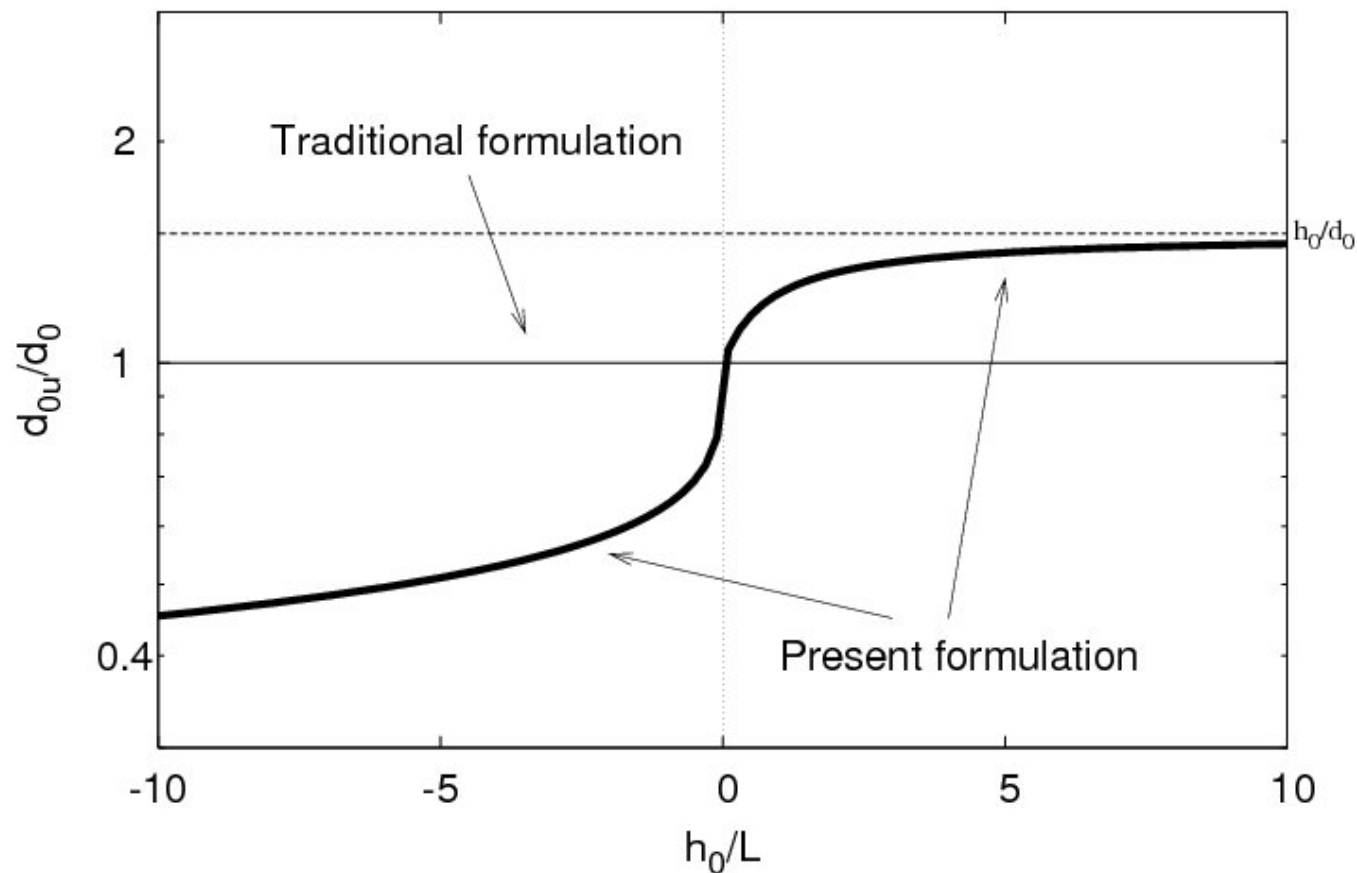


STABILITY DEPENDENCE OF THE DISPLACEMENT HEIGHT

in the "meteorological interval" $-10 < h_0/L < 10$ after new theory and experimental data

Solid line: d_{0u}/d_0 versus h_0/L

Dashed line: the upper limit: $d_0 = h_0$



Conclusions (Roughness length)

- **Traditional:** roughness length and displacement height fully characterised by geometric features of the surface
- **New:** essential dependence on hydrostatic stability especially strong in stable stratification
- **Applications:** to urban and terrestrial-ecosystem meteorology
- **Especially:** urban air pollution episodes in very stable stratification

